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PROPELLER THEORY OF PROFESSOR JOUKOWSKI AND HIS PUPILS

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PROPELLER THEORY OF PROFESSOR JOUKOWSKI AND HIS PUPILS.*

By W. Margoulis.

I will give a summary of the work done in Russia from 1911 to 1914, by Professor Joukowski and his pupils Sabinine, Iourief and Wettchinkine, which has hitherto remained but little known in other countries.

This summary will show that these men were the true originators of the theory, very widespread at the present time, which combines the theory of the wing element and of the slipstream, and which in 1914, they had already completely developed in a very original form.**

The demonstrations of these authors are very interesting but we shall confine ourselves, owing to lack of space, to indicating the definitive formulas they established and to the setting forth of the main features of their work.

We will adopt the symbols given by Mr. Wettchinkine in a remarkable treatise, "Calcul de l'Hélice Propulsive" (Calculation of the Propeller), (Bulletin de la Société Polytechnique de Moscou, 1913, No.5), in which he summarized and completed the works of Joukowski, Iourief and Sabinine. These notations are:

r, radius of any section of the propeller;

R, radius of periphery;

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* From "L'Aéronautique," August, 1921.

** A similar theory was set forth in England in 1916, by Fage and Collins, "Investigations of the Magnitude of the Inflow Velocity in the Immediate Vicinity of an Airscrew," (R & M 328), which gave rise to numerous experimental researches of the N.P.L. (R & M Nos. 460, 475 and 565).

R_o, radius of hub;

D, diameter of propeller = 2R;

W, forward speed of propeller;

w, axial component of the indraft velocity in front of the propeller, for the radius r;

w_2, axial component of the velocity of the slip stream behind the propeller, for the radius r;

$W_1 = W + w$;

Ω, angular rotary speed of propeller;

v, tangential component (in the plane of rotation) of the indraft velocity in front of the propeller, for the radius r;

v_2, tangential component of the velocity of the slip stream behind the propeller, for the radius r;

$$\Omega_1 = \Omega - \frac{v}{r}; \qquad z = \Omega \frac{r}{w} \text{ and } z_1 = \Omega_1 \frac{r}{W_1}$$

$V = \sqrt{W_1^2 + \Omega_1^2 r^2}$, resultant velocity with radius r;

K_X, K_y, coefficients of the resultant of the air stresses on the propeller;

$\mu = K_X/K_y$

b, width of propeller for the radius r.

It is easily demonstrated that the elemental lift and power are

(1) $$dP = 2bK_y \Omega_1 r W_1 \sqrt{1 + z_1^2}\, X_1 \, dr,$$

(2) $$dT = 2bK_y \Omega r W_1^2 \sqrt{1 + z_1^2}\, X_2 \, dr,$$

in which

$$X_1 = 1 - \frac{\mu}{z_1} \text{ and } X_2 = 1 + \mu z_1.$$

Sabinine and Iourieff were the first to conceive the idea of determining the stresses undergone by a section of the propeller blade, by assuming that the velocity along the axis of the propeller was equal to the speed of translation plus a certain velocity of indraft, determined by the application of the theorem of the quantity of motion. They thus combine, for the first time, W. Froude's theory of the element of the blade* with R. E. Froude's theory of the ideal propeller.**

Their works, dating from 1911, were expounded in a series of lectures on the "Basic Theories of Aeronautics," by Professor Joukowski, before the Imperial Technical School at Moscow, in 1911-1912.

The authors considered only the axial indraft velocities (w and w_2) and the propellers for which these velocities remained uniform along the radius. The relation between w and w_2 was fixed by assuming for the section, where the velocity is w_2, a coefficient of contraction equal to that of a stream running through a hole in the bottom of a tank which is 0.56. Hence $w = 0.56\ w_2$.

The velocity w was determined by making the propeller thrust equal to the amount of motion imparted to the fluid.

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* This theory is generally attributed, even in France, to Mr. Drzewiecki, but W. Froude first published his theories in 1878, fourteen years before Mr. Drzewiecki, in his memoirs, "On the Elementary Relation between Pitch, Slip and Propulsive Efficiency" (Transactions of the Institute of Naval Architects, 1878). This memoirs contained the notion of the optimum angle.

** It will be evident further on, from the authors' very original method of determining the indraft velocity, that the original work of R.E. Froude, "On the Part Played in Propulsion by Differences of Fluid Pressure" (Transactions of the Institute of Naval Architects, 1889), as well as the subsequent works of Finsterwalden were unknown to them.

(3) $$P = \pi \rho (R^2 - R_0^2) W_1 w_2 = \pi \rho (R_2 - R_0^2)(W + w) \frac{W}{0.56}$$

The shape of the blade was fixed by making the axial component of the thrust of the air on an element of the blade equal to the amount of motion. Thus, for a two-bladed propeller, we have:

$$2bK_y \Omega r W_1 \sqrt{1 + z_1^2} X_1 dr = 2 \pi \rho W_1 w_2 r dr;$$

whence

(4) $$b = \frac{\pi \rho}{K_y} \frac{w_2}{\Omega} \frac{1}{X_1 \sqrt{1 + z_1^2}}$$

The motive power is given by the integral of formula (2)

$$T = \int_{R_0}^{R} 2b K_y \Omega r W_1^2 \sqrt{1 + z_1^2} X_2 dr,$$

in which $W_1 = W + w$ is a constant determined by equation (1).

In pursuance of their work, Sabinine and Iourief, in a communication given in the above-mentioned memoir of Mr. Wetchinkine, to the second Russian Congress of Aeronautics in 1912, endeavored to establish the relation between the velocities w and w_2, by introducing the notion of the velocity of rotation in front of the propeller. By considering, as before, a propeller with constant velocity of inflow (w) and outflow (w_2), they demonstrated that the rotation velocities v and v_2 were inversely proportional to the radius.

$$v = \frac{W_1 w_2}{\Omega r}, \qquad v_2 = \frac{W_1 w_2}{\Omega r_2}$$

r_2 being the radius of the narrowest section of the stream, cor-

responding to the radius r in the plane of the propeller.

They then established the following relation between w and w_2

$$w = \frac{w^2}{2}\left\{1 + \left(\frac{W_1}{\Omega R_2}\right)^2\right\} \tag{5}$$

in which R_2 is the radius of the smallest periphery of the stream.

This relation differs but little from the one they had adopted in their first work. The determination of the velocity w, of the shape of the blade and of the power, was made the same as in their first work, which is to say that, if we encounter here for the first time the notion of the velocity of rotation, this notion is only utilized for establishing the relation $w = f(w_2)$.

Mr. Wettchinkine, in his previously cited memoir, extended this method to any propeller for which the velocity w was not uniform along the radius. For the equation $w = f(w_2)$ he obtained the following expression

$$w = \frac{w_2}{2} + \frac{1}{w_2}\left\{\frac{1}{2}\left(\frac{W_1 w_2}{\Omega r_2}\right)^2 - \int_{r_2}^{R_2}\left(\frac{W_1 w_2}{\Omega r_2}\right)^2 \frac{dr_2}{r_2}\right\}$$

the relation between r and r^2 being given by the equation of continuity

$$\pi r^2 W + \int_0^r 2\pi wr\, dr = \pi r_2^2 W + \int_0^{r_2} 2\pi w_2 r_2\, dr_2$$

The value of the velocity w_2 is fixed by equation (4), in which the author makes $X_1 = 1$, since he considers that the velocities of flow must be determined by assuming a perfect fluid.

As we shall see further along, Professor Joukowski takes the same point of view in his works.

Mr. Wettchinkine called attention to the fact that the theory of Sabinine and Iourief, as well as the generalized theory, contained an error, due to the assumption that the projection of the amount of motion on the axis was equal to the propeller thrust, while it was really equal to this thrust plus the projection, on the axis, of the forces due to the hydrodynamic pressures, of which, however, the authors took account in establishing equation (5).

Sabinine and Iourief had introduced the notion of the component of the slip stream (or inflow) velocity in the plane of the propeller, but they had not made allowance for this velocity in determining the angles of attack and the stresses undergone by the blades.

Professor Joukowski, in his "Theorie tourbillonnaire de l'helice" (Vortex Theory of the Propeller),* introduced the additional velocity of rotation into the calculation of the resultant velocities and the angles of attack.

The author assumes (Fig. 1) that the flow of the fluid around the propeller is governed by an axial vortex located behind the propeller and revolving in the same direction as the latter with a circulation of 2×1. This vortex separates into two adjoining series of vortices, such that the circulations of the velocities

* Travaux de la section Physique des Amis des sciences naturelles, Moscow, 1912, No. 1.

along the contours of the blades are equal to 1. They leave the ends of the blades in the form of two helicoidal vortices, as shown in Fig. 1.* The author demonstrates that

$$v = \frac{1}{2\pi r}; \quad \gamma_2 = 2 = \frac{1}{\pi r}$$

and that

$$w_2 = 2w.$$

He considers the case of a propeller with a constant circulation along the blade, for which the inflow and outflow velocities are uniform along the radius, the same as for the Sabinine-Lourief propeller. He then has

$$2w(W + w) = \frac{\Omega 1}{\pi}\left(1 - \frac{1}{2\pi\,\Omega\,R^2}\right), \tag{6}$$

an expression connecting w and v.

On the other hand, according to professor Joukowski's famous theorem on the value of the coefficient of lift in terms of the circulation,

$$1 = \frac{K_y}{\rho} bV = \frac{K_y}{\rho} b \sqrt{(W + w)^2 + \left(\Omega r - \frac{1}{2\pi r}\right)^2} \tag{7}$$

The expressions for thrust and power are obtained by combining equations (1), (2) and (7), when we have

$$dP = 2\rho 1 \; \Omega_1 \, rX_1 \, dr, \tag{8}$$

$$dT = 2\rho 1 \, W_1 \, \Omega \, rX_2 \, dr, \tag{9}$$

and the elemental efficiency is

* Mr. Riabouchinsky had previously indicated a similar arrangement of vortices around a propeller (See Bulletin de l'Institut Aerodynamique de Koutchino, 1913, No.4, p.81.

$$\eta_{el} = \frac{W}{W_1} \frac{\Omega_1}{\Omega} \frac{X_1}{X_2} = \frac{W}{W_1} \frac{\Omega r - \frac{1}{2\pi r}}{\Omega r} \frac{X_1}{X_2}$$

Mr. Wettchinkine designated the first term, which represents the output of a perfect propeller, by η and the second term, which shows by how much the flow obtained differs from the flow for a perfect propeller, by ηp. The product of these two factors is the hydrodynamic efficiency η_g. The third term is the mechanical efficiency η_m. We thus have

$$\eta = [\eta] \eta_p \eta_m = \eta_g \eta_m.$$

Mr. Wettchinkine called $\eta_{om} = \frac{\eta}{[\eta]}$, the relative propeller efficiency and drew attention to the analogy between $[\eta]$ and the efficiency of Carnot's cycle of a thermic engine; between ηp and the indicated output of the cycle realized in the engine; and between η_m and the mechanical efficiency, dependent on friction in the engine.

By integrating equations (8) and (9) and combining them with equations (6), we obtain the expressions for thrust, power, and efficiency in terms of l and of $Z = \frac{\Omega R}{W}$ only.

Mr. Wettchinkine made tables and graphics which rendered it possible to make rapidly the determination of the characteristics of a propeller with a constant circulation following the conditions imposed, either for propellers in flight or stationary. (Many of these propellers were tried out successfully in flight.)

Mr. Wettchinkine likewise applied Professor Joukowski's theory to every propeller for which the circulation was not constant

along the radius. Fig. 2 shows the distribution of the vortices in this case.

For these propellers, all the equations remain the same, except No.6, which becomes

$$2w\,(W + w) = \frac{\Omega}{\pi}\,1\quad\left(1 - \frac{1}{2\pi\,\Omega^2}\right) + \int_r^R \quad \frac{1^2}{\pi^2\,r^3}\,dr,$$

which is solved by successive approximations.

In 1913 and 1914 Professor Joukowski published two other memoirs on the "Vortex Theory of the Propeller"* in which he discussed the questions of auto-rotation, the influence of the number and width of the blades on the functioning of aircraft propellers and propellers for wind tunnels.

We will call attention to one more work of Mr. Wettchinkine, "Invariants of the Propeller,"* in which he proposes to designate the invariants of a propeller by

$$\frac{T}{n^2\,D^4},\quad \frac{PW}{n^3\,D^5},\ \ldots\ldots,\qquad \frac{T}{W^2\,D^2},\ \ldots\ldots,\qquad \frac{Tn^2}{W^4},\ \ldots\ldots,$$

and determines the characteristics of Professor Joukowski's propeller with constant circulation, giving the maximum output for successive values of either $\frac{P}{V^3\,D^2}$ or $\frac{Pn^2}{V^5}$.

Lastly, in an experimental work (communication to the fourth Russian Congress of Aeronautics in 1914) the same author compared the velocities and pressures measured before and behind a propeller held stationary on the ground, with the values calculated by means of Professor Joukowski's theory.

* Travaux de la section Physique de la Societé des Amis des Sciences naturelles, Moscou, 1914.

We give below (Fig. 3) for illustration, the results of measurements of the three components of the mean velocities, made by the author on a two-bladed helicopter propeller* having a diameter of 1.5 and a constant circulation. For this propeller

$$2\overline{l} = \frac{l}{\pi \Omega R^2} = 0.03;$$

at 640 t : m, the calculated thrust and torque were respectively 15.1 kg and 1.87 kgm, while the measured values were 13.2 kg and 1.75 kgm.

In the left-hand diagrams, the arrows represent, in magnitude and direction, the velocities in a plane passing through the axis of the propeller, while in the right-hand diagram the arrows represent the velocities in planes perpendicular to the radii. The small circles on the left indicate the values of the components parallel to the axis and the circles on the right indicate the components in a plane parallel to the rotation plane of the propeller.

The curves -.-.-.-.- represent the calculated values of w, $2w = w_2$, v, and vX_2. It is evident that, behind the propeller, the calculated velocities are in accord with the measured values.

From this brief review we can appreciate the importance of the work done by Professor Joukowski and his pupils on the theory of the propeller, before the war, and we regret that we are not

* It may be demonstrated by means of Prof. Joukowski's theory that the maximum thrust of a helicopter propeller, with a given diameter and power, is obtained with a propeller having a constant circulation, while the maximum efficiency of a pusher propeller, for given values of either $\frac{P}{V^3 D^2}$ or $\frac{Pn^2}{V^4}$, is obtained with a propeller having a circulation which increases toward the periphery.

informed in regard to the researches they have since been able to carry out in connection with the same problem.

Translated by the National Advisory Committee for Aeronautics.

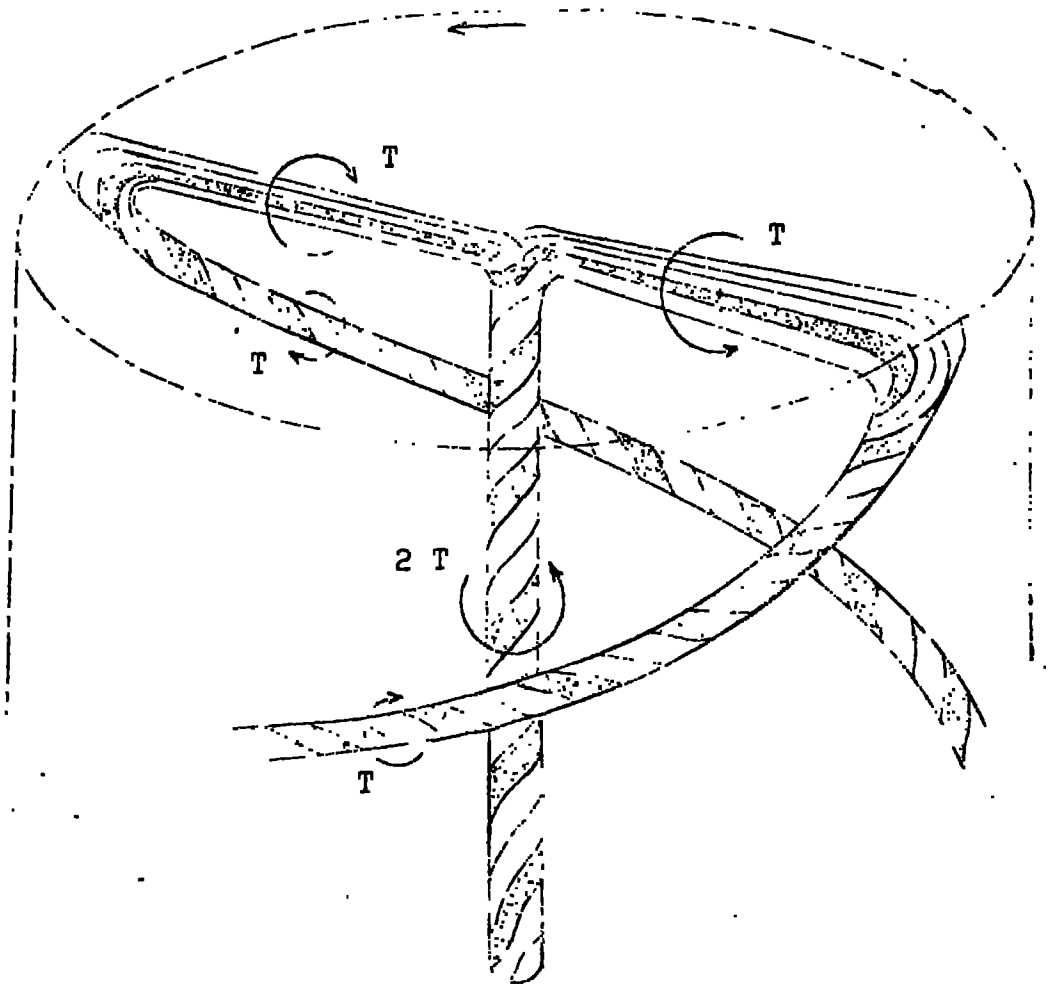

Fig.1 - Disposition of vortices in a propeller with constant circulation along the radius.